WHY 4.44?

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with special thanks to Dr. Doug Miron

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Why is 4.44 the K_f factor for flux density of a sine wave? The answer is rooted (no pun intended) in the RMS versus the average value of a voltage.

Due to our electrical engineer’s preoccupation with the effective (rms) power a sinusoidal voltage will provide, we tend to think of all voltages in terms of their RMS equivalent. The RMS equivalent is NOT, however, the same as the average value you would find if you summed up the voltage levels at each degree of its phase. (Substitute “teeny-weeny subinterval” for “degree” if you prefer).

WHY 4.44?… cont...

The difference between average and RMS is subtle, but important. The RMS (also known as “effective”) voltage value is calculated by taking the values of voltage at each subinterval, squaring them, taking the average of the squares, then taking the square root of the result (hence the term root-mean-square). The average value is simply the mean of the values taken over the interval. Note that the average value of a sinusoid is zero since it spends an equal and symmetrical amount of time above zero as it does below zero. We may prefer to think of the average voltage over a half interval to give it some meaning. We will show how the value of the average compares to the RMS value in the following slides.

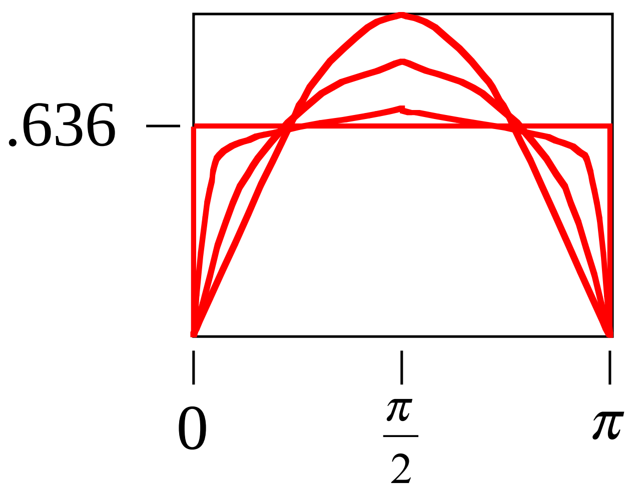
WHY 4.44?… cont...

Just to get the notion that the RMS equivalent is the one and only way to measure a sinusoidal voltage, lets talk instead in terms of pressure. Since voltage is analogous to pressure, lets consider an air tank in which the pressure is varying sinusoidally from -10 psi to +10 psi (obviously gage pressure since you can't have less than zero absolute pressure). While the peak pressures are established at +10 psi and -10 psi, the average is clearly zero. But what is average force applied to the vessel's hatch during the positive half-cycle if the hatch is exactly 100 in^2? Lets suppose, for sake of argument, that the hatch can take a lot of fast-pressure abuse, but long-term stress above its rating of 700 lbs would cause it to break loose.

WHY 4.44?… cont...

To make sure we're operating in the safe zone, we need to calculate the average pressure on the hatch. We can do this graphically or mathematically. Both approaches are shown here.

$$P = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \sin(t)\,dt$$



$$= \frac{1}{\pi - 0} \left[\cos(0) - \cos(\pi)\right]$$

$$= \frac{1}{\pi}\left[1 - (-1)\right] = \frac{2}{\pi} \approx 0.63662$$

WHY 4.44?… cont...

From this we can see that the average pressure would be 6.366 lbs/in^2 which would exert 636.6 lbs on the 100 in^2 hatch, thus meeting the safety maximum of 700 lbs.

Supposing that we had instead assumed the RMS equivalent, which is $\frac{\sqrt{2}}{2}$ or about 0.707 times the peak value. In this case we would have incorrectly assumed that the average value of pressure was 707 lbs, or about 7 lbs beyond the safe limit.

WHY 4.44?… cont...

RMS is just one way to get a ‘handle’ on a sine wave. It is just as valid to describe a sine wave in terms of its peak or average voltage. The advantage of RMS is that it tells us the equivalent DC voltage that produces the same *power* as would the sine wave. While this is great when you’re working with power, it isn’t so great when you’re dealing strictly with voltage.

Flux density is a function of voltage, *not power*, so we must consider the average value of the voltage, not the voltage that happens result in an equivalent power level.

So, where does that leave us when we need to convert a voltage into its flux density? The answer depends on whether we are starting with RMS or peak voltage. Lets start with a square wave to see how the basic formula is derived.

WHY 4.44?… cont...

Faraday's law states that the flux $\phi(t)$ inside the core induces voltage $v_{turn}(t)$ in each turn of the winding, or

$$v_{turn}(t) = \frac{d\phi(t)}{dt}$$

but $\phi(t)$ passes through each turn of the winding, so the net average winding voltage is

$$v(t) = n v_{turn}(t) = n \frac{d\phi(t)}{dt} = n A_C \frac{dB(t)}{dt}$$

(we convert from flux to flux density by taking core area A_C into account)

WHY 4.44?… cont...

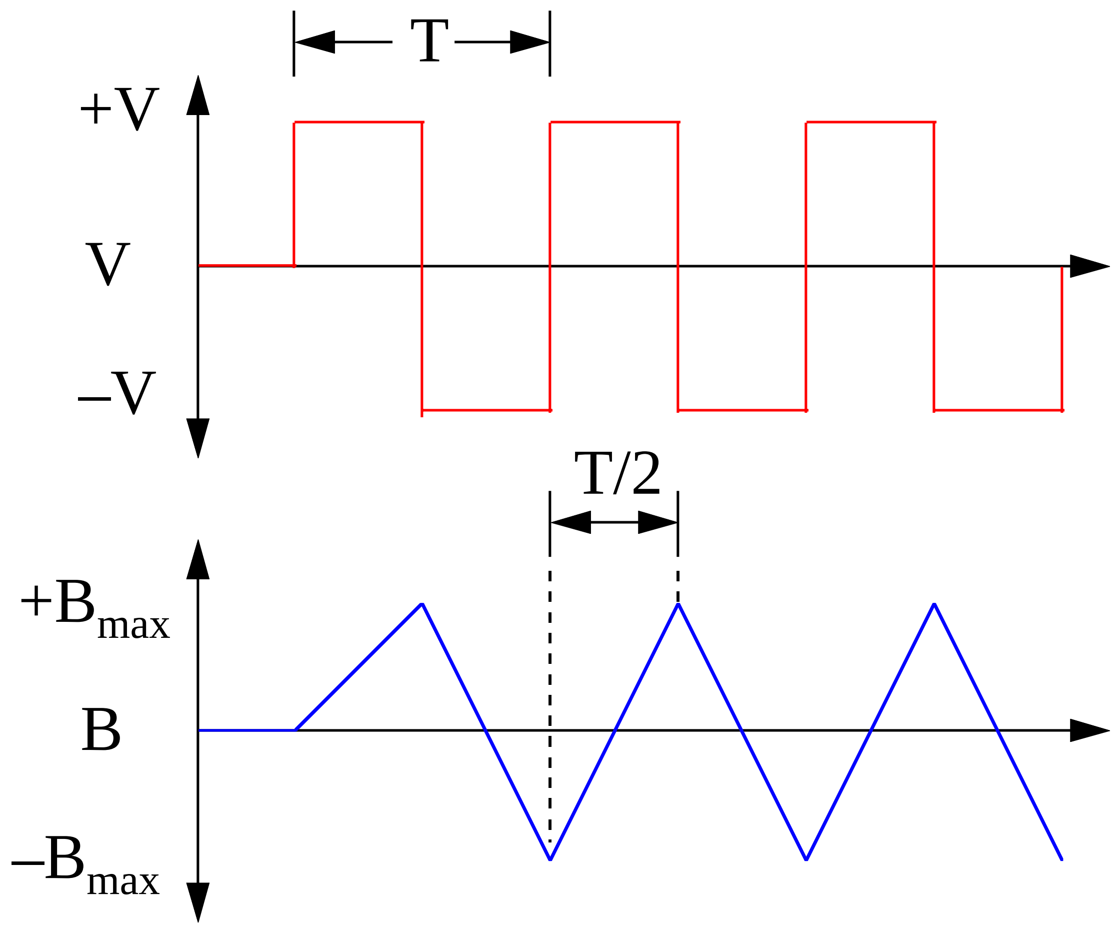
Continuing with the last part of the equation

$$v(t) = nA_C \frac{dB(t)}{dt}$$

$$= nA_C \left(\frac{\text{change in flux density}}{\text{change in time}} \right)$$

$$= nA_C \left(\text{change in flux density} \cdot \text{frequency} \right)$$

WHY 4.44?… cont...



Here we see that a square wave excitation voltage results in a triangle wave of flux in the core. The slope of the flux is thus $(B_{max}-(-B_{max}))/(T/2)=4B_{max}$.

WHY 4.44?… cont...

Since the slope of the excitation flux determines the output voltage, the average output voltage for the square wave is

$$V_{avg} = 4 \cdot B_{\max} \cdot N \cdot A_C \cdot f$$

But this is for A_C in m^2 and flux density B_{max} in teslas. If we wish to use cm^2 and gauss, we must multiply the right-hand side of the above equation by

$$\left[10^{-4} \text{ to convert m}^2 \text{ to cm}^2\right]^2 \cdot \left[10^{-4} \text{ to convert tesla to gauss}\right] = 10^{-8}$$

WHY 4.44?… cont...

The sinewave

A square wave's average value is the same as its peak value (or its RMS value, for that matter), but we typically describe a sinusoidal voltage by its RMS equivalent. We must convert from rms to average by taking a trip through the peak voltage. Since

$$V_{Peak} = \frac{V_{rms}}{\frac{\sqrt{2}}{2}} \approx \frac{V_{rms}}{0.707} \quad \text{and} \quad V_{avg} = V_{Peak} \cdot \frac{2}{\pi} \approx 0.636 \cdot V_{Peak}$$

$$\therefore \;\; V_{avg} = V_{rms} \cdot \frac{\frac{2}{\pi}}{\frac{\sqrt{2}}{2}} \approx \frac{V_{rms}}{1.11}$$

WHY 4.44?… cont...

$$V_{avg} = \frac{V_{rms}}{1.11} = 4 \cdot B_{\max} \cdot N \cdot A_C \cdot f$$

Solving for flux density, B_{max} yields

$$B_{\max} = \frac{V_{rms}}{1.11 \cdot 4 \cdot N \cdot A_C \cdot f}$$

$$B_{\max} = \frac{V_{rms}}{4.44 \cdot N \cdot A_C \cdot f}$$

Remember: this is only valid for sinusoids

WHY 4.44?… cont...

Putting this all together brings us to the final equation for flux density in terms of voltages and two of their waveforms. Since

$$V_{avg} = 4 \cdot B_{\max} \cdot N \cdot A_C \cdot f$$

then

$$B_{\max} = \frac{V_{avg}}{4 \cdot N \cdot A_C \cdot f}$$

for B_{max} is in teslas, A_C is in m^2 and V_{avg} as a half-cycle averaged value.

or

$$B_{\max} = \frac{V \cdot 10^8}{K_f \cdot N \cdot A_C \cdot f}$$

for B_{max} in gauss, A_C in cm^2 and

K_f = 4.0 for square wave, V as the peak voltage of the square wave,

K_f = 4.44 for sine wave and V expressed as the rms value of the voltage.